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AN INTEGRATED SIMULATION AND DYNAMIC PROGRAMMING APPROACH FOR DETERMINING OPTIMAL RUNWAY EXIT LOCATIONS*

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The Federal Aviation Administration and National Aeronautics and Space Administration are researching several problems targeted at improving airport capacity. Among the foremost of these problems is the issue of improving the operational use of runways. The efficiency of runway usage is dictated primarily by the runway occupancy time (ROT) which is the time that an aircraft spends on the runway or its vicinity, until a new arrival or departure can be processed on this runway. This paper considers the problem of determining the geometry and location of high speed exits on a runway to minimize the weighted ROT of a population of aircraft under various landing scenarios and frequencies of usage. Both the problem of designing a new runway and modifying an existing one are addressed. It is shown that the continuous location problem of siting runway turnoffs admits a natural finite set of candidate optimal locations. To characterize problem data and determine optimal exit locations, a simulation program integrated with a polynomial-time dynamic programming algorithm is developed. The methodology has been implemented on a personal computer, and an example is presented to illustrate the approach.
(RUNWAY DESIGN; RUNWAY OCCUPANCY TIME; OPTIMAL LOCATION; DYNAMIC PROGRAMMING; SIMULATION)

1. Introduction

In recent years, airport congestion and delay problems have received a great deal of attention due to the rapid growth of air transportation services. The delay in an airport system increases rapidly when the air transportation demand approaches the maximum capacity of that system. The current growth rate of air passengers is about 5.8% a year (*Aviation Week and Space Technology* 1989). The average delay per landing or take-off operation of an aircraft increased by 38% from 1976 to 1986, and increased by 9% in 1986 alone. The Federal Aviation Administration's Airport Capacity Enhancement Plan of 1986 estimates the cost of delay imposed on air carriers and passengers to be 5.1 billion dollars. In the same year, 15 minutes of delay were added on an average to each flight (FAA 1988). As air transportation demand continues to grow, the problem of delay and congestion is expected to be aggravated unless airport capacity increases substantially.

An airport system is divided into six components: Airspace, runway, taxiway, apron-gate, terminal building, and ground-access facility (see Figure 1). Among these components, the airspace and the runway are usually the critical components which limit the airport capacity. There are two major factors restricting the capacities of airspace and runway components: minimum separation criteria for two consecutive landing aircraft,

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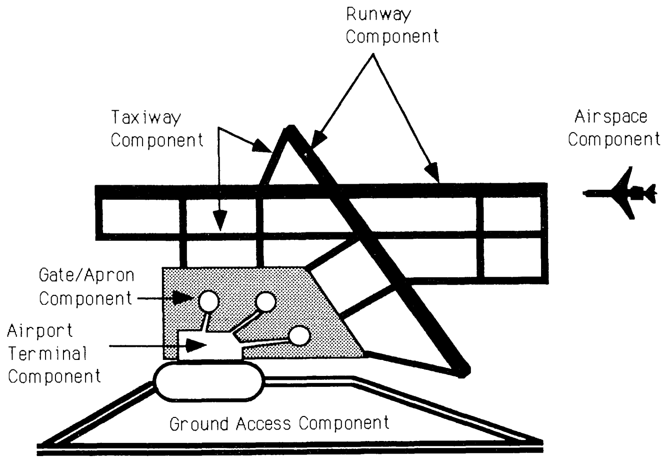

FIGURE 1. Airport System Components.

and runway occupancy time (Horonjeff and McKelvey 1983). The minimum separation criteria are imposed on the landing procedure to prevent aircraft mishap due to inadequacies in the air traffic control system, and due to the wake vortices generated by the leading aircraft. These criteria are, however, expected to decrease as a result of ongoing research efforts that address the reduction of wake vortex and the improvement of air traffic control systems (Gosling et al. 1981).

Nonetheless, for the sake of safety, an aircraft is not allowed to touch down on a runway if the leading aircraft remains on the same runway. Hence, the capacity of an airport would not increase proportionately as the minimum separation criteria decrease, unless runway occupancy time decreases too. The runway occupancy time (ROT) is defined as the time interval from the instant the landing aircraft passes over the runway threshold until it completely clears the runway (see Figure 2). This paper addresses the enhancement of runway capacity by developing a design process which minimizes the average ROT of a mixed fleet of aircraft. This is achieved via an integrated simulation and optimization approach which seeks to determine optimal locations and geometries

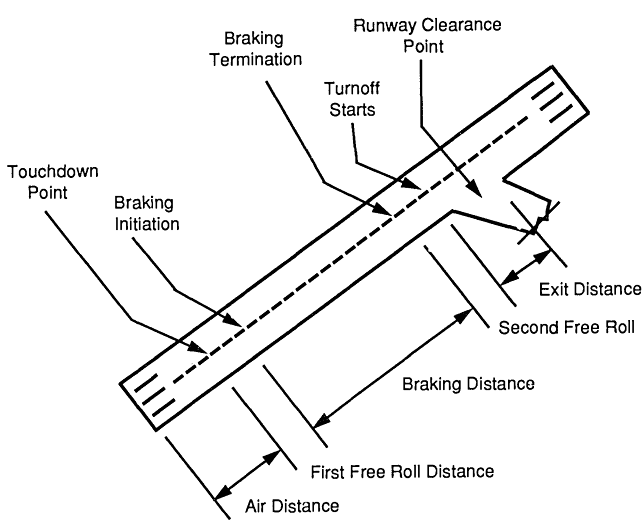

FIGURE 2. Aircraft Landing Process.

of runway exits. The inputs to this approach are the proportions of each aircraft type in the given population mix, their design exit speeds, and various airport environmental conditions such as elevation, temperature, etc.

The earliest effort to prescribe optimal exit locations is found in the work of Horonjeff et al. (1959, 1960). The purpose of their model is to determine exit locations which maximize the landing acceptance rate of a runway under the assumption of a continuous flow of aircraft according to some fixed time or distance separation rule between arriving aircraft. This is achieved by minimizing the weighted average of "wave-off" probabilities of an aircraft mix that consists of various proportions of different types of aircraft. Here, "wave-off" means that an arriving aircraft is prevented from landing because the leading aircraft is occupying the runway at the time the landing is to be made. Thus, the probability of a wave-off is the probability that the interarrival time is less than the ROT of the leading aircraft. The ROT consists of the sum of three terms: (1) time required to decelerate to the design exit speed (this will be called as a deceleration time), (2) additional time to reach the first exit after finishing deceleration, and (3) the turn-off time, which is the time interval from the beginning of the turning maneuver to the clearance of the runway. The first term depends on the aircraft type and is probabilistic in nature. The second term also varies according to the aircraft type and involves deceleration distances for each aircraft type, and is again probabilistic in nature. The third term is assumed fixed for each aircraft type. The resulting objective function (minimization of the weighted average of wave-off probabilities) is not analytically tractable, and therefore requires a numerical optimization scheme which is computationally intensive. Moreover, since the objective function involves two random variables for each aircraft, the statistical distribution information for these random variables is required. Horonjeff et al. have estimated the parameters of these distributions empirically. However, the deceleration times and distances are affected by various operational and environmental factors such as the design exit speed, aircraft landing weight, elevation, and ambient airport temperature. Hence, whenever such influencing factors change, the parameters of the distributions for the deceleration times and distances for the aircraft need to be re-estimated.

Motivated by this study, Daellenbach (1974) developed a dynamic programming model to analyze the same problem but with some extensions. Horonjeff's model imposes a strict assumption on the aircraft arrival pattern. Daellenbach relaxed this assumption, permitting a more generalized arrival pattern, thereby enhancing the computational aspect of Horonjeff's model. However, Daellenbach's model also requires the statistical distribution information for the deceleration time and distance for each aircraft as an input. The data for estimating the parameters of these distributions are difficult to collect, and are almost impossible to obtain when variations in the aforementioned influencing factors need to be considered.

In the same year, Joline (1974) developed another dynamic programming model to find the optimal number of exits and their locations with respect to the combined objective function of ROT and exit construction cost. He incorporated the ROT gain and the exit construction cost into a single objective function via a nonpreemptive approach which equated a 1-second gain in ROT with $100,000 in construction cost. While Horonjeff's and Daellenbach's models require distributions for the deceleration time and distance for each aircraft, Joline's model only needs a univariate distribution for the "ideal exit location" for a mixed aircraft population. Actually, Joline classified the aircraft into three categories according to their size, and found distributions for the ideal exit locations for these three aircraft classes based on the observations of their landing operations at Chicago's O'Hare Airport. Again, Joline's model, like the previous ones, makes the effects of the above-mentioned influencing factors hard to incorporate.

A more recent model dealing with the location of runway exits has been proposed by Tosic et al. (1985). However, this model focuses on minimizing the taxiing distance from the runway to the terminal area, and is not directly relevant to the subject of improving the runway capacity by reducing the runway occupancy time.

The runway occupancy time can be expressed as the landing time from the runway threshold to the exit location, plus the time from the beginning of the turnoff to the clearance of the runway (see Figure 2). The second term obviously varies with the geometry of the turnoff trajectory. None of the above three models accounts for this variation. The selection of the turnoff trajectory is also influenced by the design exit speed, the aircraft turning ability, and the runway surface condition. An attempt is made in the present study to accommodate these features into the model. A second point to note is that the above three models implicitly assume that an aircraft type can use more than one exit for turn-off with different ROT's and exiting probabilities. If we want to decide on the separation times between the landing aircraft based on their ROT's, it is desirable to be able to prescribe the assignment of an aircraft to some particular exit with a high probability, say 95%. Practically, this situation is expected to occur in the near future with improvements in air traffic control systems and aircraft technologies, as these will reduce the minimum separation criteria between arriving aircraft and the variability in runway occupancy time for a given exit location. Again, these features are accommodated into our model.

In contrast with the elusive data requirements for the distributions of the deceleration time and distance for each aircraft, our approach adopts a simulation model in order to estimate the parameters of these distributions for any given set of aircraft characteristics and with given airport environmental information. This simulation model considers various influencing factors such as design exit speed, aircraft landing weight, and runway surface conditions (for example wet or dry), in order to recommend exit locations and geometries that various aircraft can successfully use with high reliability. This information is used to construct an appropriate set of input data for the optimization model which then prescribes an optimal runway design via a polynomial-time dynamic programming routine.

The remainder of this paper is organized as follows. §2 lays the groundwork for constructing a model by translating a problem with a continuum of possible exit locations into an equivalent one with a discrete set of candidate locations. This influences the simulation model described in §3, which in turn provides the appropriate input for the mathematical model formulated in that section. A polynomial-time dynamic programming routine is then presented in §4 to solve both the problems of optimally designing a new runway, and for optimally modifying an existing runway. The proposed procedures are illustrated in §5 using a realistic numerical example.

2. Preliminary Modelling Constructs and Analysis

Consider the following preliminary conceptualization of the problem. Imagine a runway as being represented by the nonnegative real line, with points on it being identified by their respective distances from the origin. Suppose that there are some R combinations of types of aircraft operating under various landing scenarios (e.g., wet or dry conditions), and for each such aircraft-scenario combination r, we have (somehow) determined a range $[L_r, U_r]$ of admissible exit locations. For ease in terminology, we will refer to each such aircraft-scenario combination r as simply a distinct "*aircraft*" r. Assuming a fixed exit geometry, the left-hand or lower interval end-point L_r of the range for aircraft r might be the earliest possible exit location for which the probability of a successful turnoff is at least at some specified threshold reliability level, and the right-hand or upper interval end-point U_r might be dictated by some maximum limit on the ROT for aircraft r, for each $r = 1, \ldots, R$. Accordingly, we assume that the ROT for each aircraft r is described by some (strictly) increasing function defined on the interval $[L_r, U_r]$, for $r = 1, \ldots,$

R. Furthermore, suppose that there are a maximum of some N exits to be located on the runway at any of the continuum of points such that there is at least one exit located in each range, and that all exits are separated by at least some specified distance of $D_{\min}$. The problem addressed is to determine, if possible, a *finite* collection of points which must contain the optimal exit locations, with respect to the objective of minimizing the total weighted ROT time, where the positive weights might be determined by the frequency of runway usage. The following theorem provides a solution to this problem.

Here, given a configuration of exit locations, we will call a particular exit t_1 *superfluous* if either t_1 belongs to no aircraft range or if there exists another exit t_2 located to its left such that any range containing t_1 also contains t_2. Hence, with increasing ROT functions, no aircraft will be assigned to exit t_1, given that this is feasible, for it would rather take exit t_2 instead. Clearly, in an optimal configuration, we are interested only in *nonsuperfluous* exits.

THEOREM 1. *Assume that N is large enough so that the problem described above has a feasible solution, and suppose that we are given an optimal solution in which some $N' \leq N$ nonsuperfluous exits are located on the runway. Define a set of breakpoints as the collection of points $BP = \{L_r + qD_{\min}, q = 0, \ldots, q_r, \text{for } r = 1, \ldots, R\}$, where for each $r = 1, \ldots, R$, either $q_r = 0$, or if it exists, q_r equals the largest positive integer such that for all $q = 1, \ldots, q_r$, we have $L_r + (q - 1)D_{\min} < L_s < L_r + qD_{\min} \leq U_s$ for some $s \in \{1, \ldots, R\}$. Then each exit location in the given optimal solution coincides with a breakpoint location (see Figure 3).*

PROOF. We prove this result by induction on the exit index, with exits numbered consecutively from left to right, in a given optimal solution of N' nonsuperfluous exit locations. Consider the leftmost exit location. This exit must coincide with L_r for some $r \in \{1, \ldots, R\}$ because if not, then by sliding its location leftwards until it coincides with such a location, we will maintain feasibility (since all the aircraft which could take this exit can continue to do so), and the objective value will strictly improve. Inductively, suppose that the result is true for the location of exits $1, \ldots, t$, and consider exit $t + 1$, where $t \in \{1, \ldots, N' - 1\}$.

If exit $t + 1$ coincides with some L_r for $r \in \{1, \ldots, R\}$, then the result is true. If exit $t + 1$ is at a distance greater than $D_{\min}$ from exit t to its left, then we can slide the location

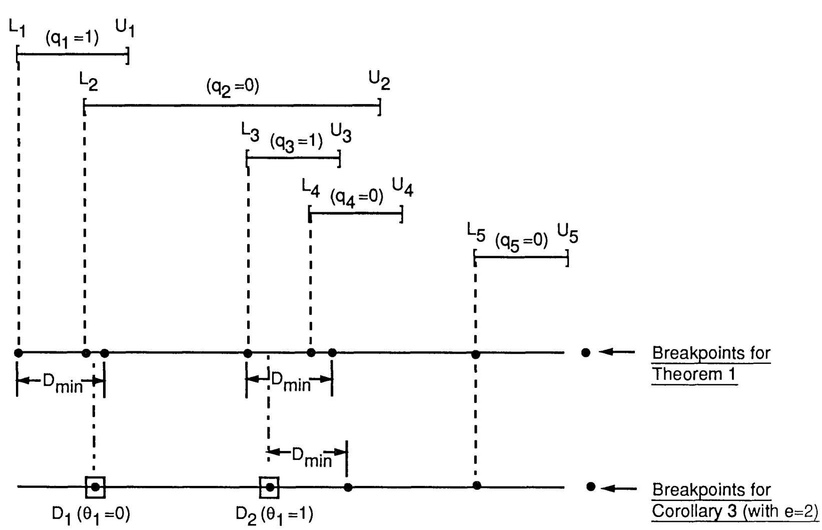

FIGURE 3. Illustration for Theorem 1 and Its Corollaries.

of exit $t + 1$ leftwards until either it again coincides with some L_r for $r \in \{1, \ldots, R\}$, or it is separated by $D_{\min}$ from exit t, thereby maintaining feasibility and improving the objective value. Hence, suppose that exit $(t + 1)$ is separated by a distance of $D_{\min}$ from exit t and that it does not coincide with any value L_r, $r \in \{1, \ldots, R\}$. Now, since exit $(t + 1)$ is nonsuperfluous, there exists some interval $s \in \{1, \ldots, R\}$ containing exit $(t + 1)$ which does not also contain exit t. Furthermore, by the induction hypothesis, exit t is located at some point $L_p + \bar{q}D_{\min}$, $0 \leq \bar{q} \leq q_p$, for some $p \in \{1, \ldots, R\}$. But by the foregoing argument, we have

$$L_p + \bar{q}D_{\min} < L_s < L_p + (\bar{q} + 1)D_{\min} \equiv \text{location of exit } (t + 1) \leq U_s.$$

Hence, by the definition of q_p, we must have $\bar{q} + 1 \leq q_p$, and so, the location of exit $(t + 1)$ is also a defined breakpoint. Consequently, the result holds true for the location of exit $t + 1$, and this completes the proof. □

COROLLARY 1. *For any pair of consecutive exits t and t* + 1 *separated by a distance greater than* $D_{\min}$ *at optimality in Theorem* 1, *the location of exit t* + 1 *must lie in* $\{L_1, \ldots, L_R\}$.

PROOF. Evident from the proof of Theorem 1. □

COROLLARY 2. *Given that the ROT's are nondecreasing within each range, rather than strictly increasing, there exists an optimal solution in which the exit locations coincide with the defined breakpoints.*

PROOF. Evident from the proof of Theorem 1. □

COROLLARY 3. *Redesign Problem. Consider a modification of the above problem in which we are also given some fixed existing exit locations at points* $D_1, \ldots, D_e$, *each lying in at least one range in* $\{1, \ldots, R\}$. *Let* $\overline{BP}$ *denote those breakpoints from BP defined in Theorem* 1 *which are separated by a distance of at least* $D_{\min}$ (*on either side*) *from any fixed exit location* $D_1, \ldots, D_e$. *Furthermore, for each* $i = 1, \ldots, e$, *let* θ_i *be the largest positive integer, if it exists, such that* $D_i + (\theta - 1)D_{\min} < L_s < D_i + \theta D_{\min} \leq U_s$ *for some* $s \in \{1, \ldots, R\}$, *for each* $\theta = 1, \ldots, \theta_i$. *Define an additional set of breakpoints* $BP_e = \{D_i + \theta D_{\min}, \theta = 1, \ldots, \theta_i$, *whenever* $\theta_i \geq 1$ *exists, for* $i = 1, \ldots, e\}$. *Then, again, any optimal solution will have the new exit locations coinciding with the resulting set of breakpoints* $\overline{BP} \cup BP_e$ (*see Figure* 3).

PROOF. The proof can be constructed similar to that of Theorem 1 by inductively considering the *new* exit locations from left to right, and arguing that if the result holds for the leftmost t new exit locations ($t \geq 0$), then since exit $(t + 1)$ is nonsuperfluous and is feasible with respect to the $D_{\min}$ separation constraint, this exit either coincides with some L_r, $r \in \{1, \ldots, R\}$, or as in Theorem 1, it is separated by $D_{\min}$ from the exit (new or existing) located to its left. Now, arguing as in Theorem 1, it can be shown that the location of exit $(t + 1)$ lies in $\overline{BP} \cup BP_e$, and this completes the proof. □

REMARK 1. By Corollary 1, or by the definition of breakpoints in Theorem 1, it is evident that if $D_{\min}$ is small enough, the optimal exit locations will coincide with the points L_r, $r = 1, \ldots, R$. This can be seen by noting from the proof of Theorem 1 that the leftmost exit location must coincide with L_r for some $r \in \{1, \ldots, R\}$. Now, if the distance between successive L_r values exceeds $D_{\min}$, then since the next nonsuperfluous exit location must lie in some new range to the right, and therefore be separated from the previous location by a distance exceeding $D_{\min}$, it will again coincide with some L_r value by Corollary 1. For larger values of $D_{\min}$, the other breakpoints will begin to play a role. This is of consequence, since, in the sequel, the points L_r will turn out to be the critical exit locations as determined by the simulation runs.

To summarize, Theorem 1 prescribes all the potential exit locations which effectively contain an optimal solution to the continuous exit location problem. Corollary 3 pertains to the redesign problem in which some fixed, existing exit locations are prespecified.

3. Data Characterization via Simulation: Mathematical Model Construction

Suppose that we are given $r = 1, \ldots, R$ aircraft types operating under various landing scenarios. As in the previous section, let us simply refer to each such aircraft-scenario combination as "aircraft" r, for $r = 1, \ldots, R$. Denote by $w_r > 0$ the relative frequency of runway usage for aircraft r, for $r = 1, \ldots, R$, where $\sum_{r=1}^{R} w_r = 1$. This quantity might be taken as the product of the proportion of actual aircraft which are of the particular type times the probability of occurrence of the scenario in the corresponding aircraft-scenario combination. The first task is to prescribe a lower interval end-point L_r for the range of admissible exit locations for each aircraft $r = 1, \ldots, R$.

Toward this end, we use a simulation model which divides the landing sequence into five phases. (See Hobeika et al. 1990 for details of this model.) In this landing sequence, after passing the runway threshold, the aircraft first flies over the runway to touchdown with air drag deceleration (phase 1). The aircraft then coasts on the runway for a few seconds with a constant speed (phase 2). Next, the pilot begins to decelerate until the speed declines to the prespecified design exit speed (phase 3). Following this, the aircraft coasts on the runway for a few seconds at the design exit speed before taking a turnoff (phase 4). Finally, the aircraft begins to turn and exits following a prescribed exit geometry (phase 5). The ROT of the aircraft is the time passed from the runway threshold to the clearance of the runway. The simulation model calculates the mean and standard deviation of the location where the aircraft finishes phase 4, and calculates the mean and standard deviation of the time when the aircraft finishes phase 5.

Using the foregoing output information and assuming a normal distribution for the location, we prescribe a recommended ideal exit location L_r based on a specified reliability ρ of making a successful turnoff, and, accordingly, we compute an estimated ROT value T_r as

$$L_r = \mu_r^L + \phi_\rho \sigma_r^L \quad \text{and} \quad T_r = \mu_r^T + \phi_\rho \sigma_r^T \quad \text{for} \quad r = 1, \ldots, R, \tag{1}$$

where μ_r^L and μ_r^T are, respectively, the mean turnoff location and ROT for aircraft r, σ_r^L and σ_r^T are the respective standard deviations for the turnoff location and the ROT for aircraft r, and ϕ_ρ is the ρ-percentile value for the standard normal distribution.

Based on the above set of primary exit locations (and exit geometries) as determined by the execution of the simulation runs for each $r = 1, \ldots, R$, and given a minimum exit separation distance of $D_{\min}$, we employ Theorem 1 (along with Corollary 3 if applicable) to determine the secondary set of potential exit locations, with certain prescribed exit geometries. (The FAA recommends that the minimum separation distance $D_{\min}$ be at least 229 m or 750 ft., although there is some flexibility in selecting this parameter value.) Hence, for example, with 3 aircraft types under 2 landing pavement condition scenarios, say, wet and dry, if the $R = 6$ combinations yield L_r values as {1000, 1100, 1300, 1400, 1600, and 1700} meters, then by Theorem 1, assuming $U_r = 2{,}000$ m $\forall r = 1, \ldots, R$, the continuous optimal exit location problem would determine optimal exit locations confined to the discrete set of locations {1000, 1100, 1229, 1300, 1329, 1400, 1458, 1529, 1558, 1600, 1629, 1687, 1700, 1758, 1787, 1829, 1858, and 1916} meters, given $D_{\min} = 229$ m. Let us number the set of potential exit locations consecutively from left to right as $k = 1, \ldots, K$, and let us denote by $l(k)$ the location (distance) of exit k down the runway for $k = 1, \ldots, K$.

The next step is to compute the ROT value T_{rk} for each aircraft $r = 1, \ldots, R$ and each potential exit location $k = 1, \ldots, K$. Denote by $k(r)$ the potential exit index corresponding to the location L_r on the runway, for each $r = 1, \ldots, R$. Then, for each $r = 1, \ldots, R$, if $k < k(r)$, we arbitrarily set $T_{rk} \equiv 0$ and note that this exit is inadmissible for aircraft r. For $k = k(r)$, we use $T_{rk} = T_r$ as given by equation (1). For $k > k(r)$, we compute

$$T_{rk} = T_r + T_{rk}^{\text{free}} + T_{rk}^{\text{off}}. \tag{2}$$

The time T_{rk}^{free} is the free roll time (without braking) taken by the aircraft while decelerating at the rate gf, from the exit speed V_r, and then cruising at a fixed user-specified taxiing speed V_r^{tax}, if and when this speed is reached. Here, $g = 9.81$ m/sec^2 is the gravitational acceleration rate and $f = 0.03$ is the rolling friction coefficient. Noting that the distance travelled before reaching taxiing speed is given by

$$D_{rk}^{\text{tax}} = [V_r^2 - (V_r^{\text{tax}})^2]/2gf \tag{3a}$$

and denoting the distance to be travelled to exit k from the (ideal) exit $k(r)$ as $D_{rk} = l(k) - l[k(r)] \equiv l(k) - L_r$, where L_r is given by equation (1), we have

$$T_{rk}^{\text{free}} = \begin{cases} \dfrac{D_{rk}}{(V_r + V_{rk}^{\text{term}})/2} & \text{if} \quad D_{rk} \le D_{rk}^{\text{tax}}, \\ \dfrac{D_{rk}^{\text{tax}}}{(V_r + V_r^{\text{tax}})/2} + \dfrac{(D_{rk} - D_{rk}^{\text{tax}})}{V_r^{\text{tax}}} & \text{if} \quad D_{rk} > D_{rk}^{\text{tax}}, \end{cases} \tag{3b}$$

where V_{rk}^{term} is the terminal exit speed given by $V_{rk}^{\text{term}} = \sqrt{V_r^2 - 2gfD_{rk}}$ in case $D_{rk} \le D_{rk}^{\text{tax}}$.

The third term T_{rk}^{off} in equation (2) is intimately related to the turnoff trajectory or geometry, and its exact calculation requires a computationally intensive numerical integration process. Since this turnoff time is typically lesser than 25% of the total ROT, we adopt an approximation scheme described in Hobeika et al. (1990) to estimate it. This scheme is based on a 2-centered circular compound curve approximation to the exit trajectory, and assumes a natural rolling friction deceleration rate. Usually, T_{rk}^{off} ranges from 6–13 secs, according to the size of the aircraft, its design exit speed, and the desired exit angle. We remark here that once T_{rk} exceeds the user-specified tolerance for the maximum permissible ROT for some $k = k_1$, we set the upper range interval end-point for aircraft r as $U_r = l(k_1 - 1)$, and we let $T_{rk} = \infty$ for $k \ge k_1$.

A mathematical formulation of our problem can now be constructed as follows. Define the set of admissible exit locations for aircraft r as

$$A(r) = \{k \in \{1, \ldots, K\}: \quad l(k) \in [L_r, U_r]\} \qquad \text{for each} \qquad r = 1, \ldots, R. \tag{4}$$

Furthermore, progressing from left to right down the runway and examining sets of locations that lie within an interval of length $D_{\min}$, construct all possible sets of mutually exclusive candidate locations S_t, $|S_t| \ge 2$, $t = 1, \ldots, \tau$, say, such that the $D_{\min}$ separation constraints are satisfied if and only if at most one exit is located in each such set. Then, defining the binary variables,

$$x_k = \begin{cases} 1 & \text{if an exit is located at site } k \\ 0 & \text{otherwise} \end{cases} \qquad \text{for} \qquad k = 1, \ldots, K, \tag{5a}$$

$$y_{rk} = \begin{cases} 1 & \text{if aircraft } r \text{ is assigned to exit } k \\ 0 & \text{otherwise} \end{cases}$$

$$\text{for} \qquad k \in A(r), \qquad r = 1, \ldots, R, \tag{5b}$$

we may formulate the problem of minimizing the total weighted runway occupancy time (WROT) as follows.

$$\textbf{WROT}: \text{ Minimize} \quad \sum_{r=1}^{R} \sum_{k \in A(r)} w_r T_{rk} y_{rk} \tag{6a}$$

$$\text{subject to} \quad \sum_{k \in A(r)} y_{rk} = 1 \quad \text{for each} \quad r = 1, \ldots, R, \tag{6b}$$

$$\sum_{k \in S_t} x_k \leq 1 \quad \text{for each} \quad t = 1, \ldots, \tau, \tag{6c}$$

$$\sum_{k=1}^{K} x_k \leq N, \tag{6d}$$

$$y_{rk} \leq x_k \quad \text{for each} \quad k \in A(r), \quad r = 1, \ldots, R, \tag{6e}$$

$$x \text{ and } y \text{ are binary vectors.} \tag{6f}$$

The objective function (6a) represents the aggregate expected runway occupancy time. Constraint (6b) requires that each aircraft type should be assigned to one (available) exit under each scenario. Constraint (6c) ensures a feasible mix of exits, while constraint (6d) enforces a maximum limit on the total number of exits constructed. The constraints (6e) enforce that only the constructed exits are used, and constraints (6f) represent the logical restrictions on the variables.

REMARK 2. Several points are worth noting at this junction. First, the same formulation given above may be used to model the problem of redesigning or modifying existing runways, by simply fixing the appropriate variables x_k to be one. This option can also be adopted for *a priori* enforcing the choice of certain exits. Second, note that the value of N is assumed to be selected in the range $[1, \min\{R, K\}]$, such that the problem is feasible. (By usual practice, a right-angle, default exit is usually provided at the end of the runway for worst-case scenarios.) The actual value of N used might be dictated by other regulatory and budgetary considerations, and the user might wish to investigate the sensitivity of the model solution to various values of N. Third, we can accommodate choices for various exit geometries, some even at the same location, by treating each such choice as a distinct exit in the ordered set of exits $k = 1, \ldots, K$. Fourth, the above model represents the problem for aircraft that are landing from one end of the runway. A symmetric problem can be solved for aircraft landing from the other end, given the appropriate partitioned data for the two cases. Fifth, observe that Problem WROT has several specialized structures including assignment, set packing, and generalized and variable upper bounding types of constraints. In fact, since with x binary, we automatically obtain y to be binary valued if (6f) is relaxed to let y_{rk} be continuous on $[0, 1]$ for each $k \in A(r)$, $r = 1, \ldots, R$, we may accordingly treat WROT as a zero-one mixed-integer linear programming problem. However, as shown next, Problem WROT can be solved in polynomial-time by a dynamic programming algorithm. This is computationally welcome, particularly if there are many combinations of aircraft types and scenarios, and of potential exit locations and geometries to be considered.

4. A Polynomial-Time Dynamic Programming (DP) Algorithm

We now present a dynamic programming (DP) formulation or decomposition for Problem WROT. To facilitate this, let us construct an imaginary candidate $k = 0$ at the point $l(0) = -D_{\min}$ on the runway, where we assume that $l(1) \equiv 0$. The corresponding ROT values T_{r0} are set at zero for all $r = 1, \ldots, R$, implying that no aircraft is permitted to take exit 0. Hence, there exists at least one $k \in \{0, 1, \ldots, K\}$ for which $T_{rk} = 0$ for each $r = 1, \ldots, R$. Also, for each $k = 1, \ldots, K$, let us define by $S(k)$ the set of exit locations which become invalid for selection due to the $D_{\min}$ separation constraints (6c) whenever an exit is sited at location k. Then, in order to accommodate the selection from various exit geometries at the same and at nearby locations within the DP framework, we simply need to assume that

$$T_{rk} \le T_{rk'} \quad \text{for each} \quad k' > k, \quad k' \notin S(k),$$
$$\text{for} \quad k = 0, 1, \ldots, K-1, \quad \text{and} \quad r = 1, \ldots, R. \quad (7)$$

With the foregoing definitions, the stages, states, and decisions for the DP formulation can be defined as follows, adopting the standard notation and terminology as in Hillier and Liebermann (1986), for example. (In the following description, the candidate exit k' is said to lie to the "right" of k whenever $k' > k$.)

Stage n for $n \in \{1, \ldots, N\}$, corresponds to a situation in which $(N - n)$ exits are assumed to have been constructed from the left of the runway, not including the exit at the imaginary location 0, and up to n exits can be located to the right of the last exit already located. *Stage* 0 is a dummy terminal stage.

The *states* s_n at stage n represent the possible values of the rightmost (highest index) exit currently located. For stage $n = N$, we have $s_N \equiv 0$. For any other stage $n \in \{1, \ldots, N-1\}$, the values of s_n range from $k_n, \ldots, K$ where, noting Theorem 1,

$$k_n = \max\{N - n, \min\{k \in \{1, \ldots, K\} : l(k) \ge (N-n)D_{\min}\}\}. \quad (8)$$

(Note that we can assume that $N \in [1, \min\{R, K\}]$ such that the problem is feasible as in Remark 2, and that $l(K) \ge ND_{\min}$, or else, we can accordingly alter the value of N.)

Finally, at any stage n and state s_n, the *decisions* d_n correspond to the index of the next exit to be constructed to the right of s_n. Let "$d_n = 0$" mean that no more exits are constructed. Then the possible values of d_n are 0, and $K_n, \ldots, K$, where for $n = 1, \ldots, N-1$, K_n is the smallest exit index such that $l(K_n) - l(k_n) \ge D_{\min}$, if it exists, and for $n = N$, we have $K_N \equiv 1$.

Given any stage n and state s_n, all aircraft r for which $T_{rs_n} > 0$ would have been assigned to some existing exit due to (7). Hence, the problem decomposes into locating up to n more exits to the right of s_n while satisfying the minimum separation constraint, considering only those aircraft r for which $T_{rs_n} = 0$, which implies that aircraft r has not yet been assigned to any exit. Since the optimum of this decomposed problem is independent of the previous decision, and depends only on n and s_n, Bellman's Principle of Optimality holds, and so the DP application is valid.

With the stages, states, and decisions as defined above, we can now formulate the immediate return (cost) function $c_n(s_n, d_n)$ for aircraft assigned to exit d_n, the stage transition function $T_n(d_n)$, the state transition function $s_{T_n(d_n)}$, and the backward recursion formula for the optimal accumulated return function $f_n^*(s_n)$ as given below:

$$c_n(s_n, d_n) = \begin{cases} \infty & \text{if} \quad l(d_n) - l(s_n) < D_{\min} \quad \text{and} \quad d_n \ne 0, \\ \sum_{\{r: T_{rs_n}=0, T_{rd_n}>0\}} [w_r T_{rd_n}] & \text{if} \quad l(d_n) - l(s_n) \ge D_{\min} \quad \text{and} \quad d_n \ne 0, \\ 0 & \text{if} \quad d_n = 0, \end{cases} \quad (9)$$

$$T_n(d_n) = \begin{cases} (n-1) & \text{if} \quad d_n \ne 0, \\ 0 & \text{if} \quad d_n = 0, \end{cases} \quad (10)$$

$$s_{T_n(d_n)} \equiv \tau_n(s_n, d_n) = \begin{cases} d_n & \text{if} \quad d_n \ne 0, \\ s_n & \text{if} \quad d_n = 0, \end{cases} \quad (11)$$

$$f_n^*(s_n) = \underset{d_n}{\text{minimum}} \{c_n(s_n, d_n) + f^*_{T_n(d_n)}(\tau_n(s_n, d_n))\}. \quad (12a)$$

$$f_0^*(s_0) = \begin{cases} \infty & \text{if} \quad T_{rs_0} = 0 \quad \text{for any} \quad r \in \{1, \ldots, R\}, \\ 0 & \text{otherwise.} \end{cases} \quad (12b)$$

Beginning with stage 1, at any stage n of the DP algorithm, for each possible state s_n, we find $f_n^*(s_n)$ via (12) by enumerating over all possible values of the decisions d_n. The corresponding optimal decision d_n^*, say, is stored along with the value of $f_n^*(s_n)$. At the final stage N, when all the exit locations are free, the value of $f_N^*(s_N) \equiv f_N^*(0)$ gives the optimal value of Problem WROT, and the optimal decisions can then be traced by the standard reverse scanning of the stages. Each aircraft is accordingly assigned to the selected exit for which it has the minimum ROT. Since the effort at each stage involves $O(RK^2)$ computations, the algorithm is of polynomial complexity $O(NRK^2)$.

REMARK 3. *Algorithmic Modifications for the Redesign Problem.* The DP algorithm can be readily modified to accommodate fixed, existing exit locations as treated by Corollary 3. Note that once an existing exit can be taken by an aircraft r with the stated reliability, we can let its upper range end-point U_r coincide with this location, and disregard (set at ∞) the computation of T_{rk} for higher exit indices k. In any case, we assume that, for the mix of existing and new exits $1, \ldots, K$, equation (7) holds. We then define the stages, states, and decisions precisely as before, but all with respect to *new* exit locations. The immediate return function (9) and the state transition function (11) get modified as follows, where Ex denotes the set of existing exit locations. The remainder of the algorithm stays the same.

$$c_n(s_n, d_n) = \begin{cases} \infty \quad \text{if} \quad l(d_n) - l(s_n) < D_{\min}, \quad d_n \neq 0, \\ \displaystyle\sum_{\{r: T_{rs_n}=0, T_{rd_n}>0\}} w_r \underset{\{k \in (s_n, d_n) \cap Ex \cup \{d_n\}: T_{rk}>0\}}{\text{minimum}} (T_{rk}) \\ \qquad\qquad \text{if} \quad l(d_n) - l(s_n) \geq D_{\min}, \quad d_n \neq 0, \\ \displaystyle\sum_{\{r: T_{rs_n}=0\}} w_r \underset{\{k \in Ex, k > s_n: T_{rk}>0\}}{\text{minimum}} (T_{rk}) \quad \text{if} \quad d_n = 0, \end{cases}$$

$$s_{T_n(d_n)} \equiv \tau_n(s_n, d_n) = \begin{cases} d_n & \text{if} \quad d_n \neq 0, \\ \text{maximum } \{s_n, k \in Ex\} & \text{if} \quad d_n = 0. \end{cases}$$

5. Illustrative Examples

Consider the following aircraft mix pertaining to the Oakland International Airport with frequencies as given in parentheses: EMBRAER 120 (12.5%), SHORTS 360 (3.5%), B 727 (17.5%), B 737 (28.6%), B 757 (3.2%), B 767 (5.3%), DC 9 (9.6%), BAe 146 (9.9%), DC 10 (6.5%), and B 747 (3.4%). These data are extracted from a FAA document, ignoring the aircraft types whose frequencies are less than 5000/yr (about 3%) (see FAA 1990). The desired exit speed for the first two aircraft is assumed to be 25 m/sec (56 MPH), and that of the others is assumed to be 30 m/sec (67 MPH). The exit reliability used in equation (1) is 90%. Two possible scenarios pertaining to equally likely wet and dry runway surface conditions are considered. Hence, we have $R = 20$ aircraft-scenario combinations or effective "aircraft" in the problem. For each of these combinations, we ran the simulation model to determine the 20 primary exit locations according to (1) as {1033, 1112, 1247, 1301, 1324, 1426, 1504, 1556, 1619, 1649, 1679, 1682, 1717, 1737, 1789, 1821, 1870, 1891, 2140, 2360} meters. Using $D_{\min} = 229$ m, this gives $K = |BP| = 86$ potential (primary and secondary) exit locations via Theorem 1.

Design of a New Runway

We first executed the DP algorithm assuming that a new runway is to be designed with a maximum of $N = 2$, 3, or 4 exits. The solutions for these three cases are given in Table 1, with the corresponding total optimal weighted ROT's being 48.6, 44.7, and 42.7 seconds, respectively. For example, when $N = 4$, the Boeing B727 aircraft will use the second exit

TABLE 1

Solutions for the Design Problem

(a) 2 exits case: Avg ROT = 48.6 seconds

Exit No.: Location (m):	1 1682	2 2360
Assigned Aircraft (ROT)	EMB 120-D (50.5) EMB 120-W (49.7) SHORT 360-D (57.5) SHORT 360-W (57.2) BAe 146-D (44.0) BAe 146-W (43.8) B 727-D (39.5) B 737-D (40.4) B 737-W (39.7) DC 9-D (39.0) B 757-D (40.8) B 757-W (40.0)	B 767-D (63.1) B 767-W (62.3) B 727-W (62.7) DC 9-W (62.1) B 747-D (60.0) B 747-W (58.1) DC 10-D (63.2) DC 10-W (62.4)

(b) 3 exits case: Avg ROT = 44.7 seconds

Exit No.: Location (m):	1 1649	2 1891	3 2360
Assigned Aircraft (ROT)	EMB 120-D (49.4) EMB 120-W (48.6) SHORT 360-D (56.3) SHORT 360-W (56.1) BAe 146-D (43.1) BAe 146-W (42.9) B 727-D (38.6) B 737-D (39.5) B 737-W (38.8) B 757-D (39.9)	B 767-D (48.3) B 767-W (47.1) B 727-W (47.7) DC 9-D (48.1) DC 9-W (47.1) B 757-W (49.2) DC 10-D (48.3) DC 10-W (47.3)	B 747-D (60.0) B 747-W (58.1)

(c) 4 exits case: Avg ROT = 42.7 seconds

Exit No.: Location (m):	1 1324	2 1682	3 1911	4 2360
Assigned Aircraft (ROT)	EMB 120-D (38.1) SHORTS 360-D (44.6) SHORTS 360-W (44.2) BAe 146-D (33.8) BAe 146-W (33.5)	EMB 120-W (49.7) B 727-D (39.5) B 737-D (40.4) B 737-W (39.7) DC 9-D (39.0) B 757-D (40.8) B 757-W (40.0)	B 767-D (48.8) B 767-W (47.7) B 727-W (48.3) DC 9-W (47.7) DC 10-D (48.9) DC 10-W (47.8)	B 747-D (60.0) B 747-W (58.1)

Legend: D, W: under dry and wet conditions, respectively.

located at 1682 m on the runway with a ROT of 39.5 seconds under dry conditions, and will use the third exit located at 1911 m on the runway with a ROT of 48.3 seconds under wet conditions. In either case the probability of making a successful exit is at least equal to the specified minimum reliability of 90%. The problem with $N = 4$ takes about 50 seconds of run time when implemented on an IBM PS/2 Model 50 personal computer.

Redesign of an Existing Runway

Consider now the problem of enhancing the capacity of an existing runway by adding a few new exits. Runway 28L of the San Francisco International Airport has exits located at 1585 m, 1920 m, 2315 m, and 2625 m. The first exit follows the FAA standard 30°-angled geometry. The second exit follows a 45°-angled geometry, while the other two exits are 90°-angled (see FAA 1989). The desired exit speeds for the 30°-angled and the 45°-angled exits are assumed to be 26.8 m/sec (60 MPH) and 17.9 m/sec (40 MPH), respectively, according to FAA recommendations. The exit speed for the 90°-angled exit is set to 10 m/sec (22 MPH), which is a slow enough speed for an aircraft to make a right-angle turn. Suppose now that the exit located at 1920 m is closed, and we desire to locate a new high speed exit somewhere between the first and the third exits. The total number of candidate exit locations for this case turns out to be $K = 29$ from Corollary 3.

The effectiveness of the existing runway configuration with 4 exits can be evaluated with respect to ROT using the simulation model. Table 2(a) shows the evaluation results.

TABLE 2

Solutions for the Re-Design Problem

(a) 4 existing exits: Avg ROT = 52.2 seconds

Exit No: Location (m): Type:	1 1585 30°	2 1920 45°	3 2315 90°	4 2625 90°
Assigned Aircraft (ROT)	EMB 120-D (48.7) EMB 120-W (47.3) SHORTS 360-D (57.3) SHORTS 360-W (56.3) BAe 146-D (48.7) BAe 146-W (47.5) B 737-D (43.1) B 737-W (41.3) B 757-D (43.3)	B 767-D (57.5) B 727-D (60.6) B 727-W (54.2) DC 9-D (59.1) DC 9-W (52.9) B 757-W (58.9) DC 10-D (58.3)	B 767-W (76.7) DC 10-W (77.5)	B 747-D (81.7) B 747-W (72.2)

(b) 3 existing exits and 1 additional exit: Avg ROT = 47.6 seconds

Exit No.: Location (m): Type:	1 1585 30°	2 1891 new	 1920 45°	3 2315 90°	4 2625 90°
Assigned Aircraft (ROT)	EMB 120-D (48.7) EMB 120-W (47.3) SHORTS 360-D (57.3) SHORTS 360-W (56.3) BAe 146-D (48.7) BAe 146-W (47.5) B 737-D (43.1) B 737-W (41.3) B 757-D (43.3)	B 767-D (48.3) B 767-W (47.1) B 727-D (48.6) B 727-W (47.7) DC 9-D (48.1) DC 9-W (47.1) B 757-W (49.2) DC 10-D (48.3) DC 10-W (47.3)	C L O S E D	(Unused)	B 747-D (81.7) B 747-W (72.2)

(c) 3 existing exits and 2 additional exits: Avg ROT = 45.0 seconds

Exit No.: Location (m): Type:	1 1324 new	2 1585 30°	3 1891 new	 1920 45°	4 2315 90°	5 2625 90°
Assigned Aircraft (ROT)	EMB 120-D (38.1) SHORTS 360-D (44.6) SHORTS 360-W (44.2) BAe 146-D (33.8) BAe 146-W (33.5)	EMB 120-W (47.3) B 737-D (43.1) B 737-W (41.3) B 757-D (43.3)	B 767-D (48.3) B 767-W (47.1) B 727-D (48.6) B 727-W (47.7) DC 9-D (48.1) DC 9-W (47.1) B 757-W (49.2) DC 10-D (48.3) DC 10-W (47.3)	C L O S E D	(Unused)	B 747-D (81.7) B 747-W (72.2)

Legend: D, W: under dry and wet conditions, respectively.

Here, for example, the fourth exit is declared as the only exit for B747 with ROTs of 81.7 seconds and 72.2 seconds under dry and wet conditions, respectively. The total average ROT for the existing configuration is 52.2 seconds. If we close the exit located at 1920 m and add a new high-speed exit, the average ROT will be reduced to 47.6 seconds. This can be reduced to 45.0 seconds with two additional high-speed exits. The corresponding exit locations with the aircraft assignments are given in Table 2.

To summarize, we have described in this paper an integrated simulation and optimization approach for the runway design or redesign / improvement problem. Examining the problem with a continuum of possible exit locations, we have (polynomially) prescribed a finite discrete set of potential exit locations which must contain an optimal solution. A simulation approach which incorporates appropriate approximation and extrapolation devices has been suggested for constructing the required data. These data serve as an input into a polynomial-time dynamic programming algorithm which determines the optimal (new) exit locations so as to minimize the total weighted runway occupancy time. The viability of the approach and the sensitivity of the results to the number of exits is illustrated on a small but realistic example implemented on a personal computer. The effort for larger problems can be estimated by extrapolation using the $O(NRK^2)$ complexity formula. Also, it is quite conceivable that this analysis and methodology can be used in other application contexts as well such as in multiprocessor batch scheduling of jobs, given time window constraints for the jobs, and a limit on the number of fixed set-ups for the system.[1]

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